

# Optimal Dynamic Resource Allocation for Multi-Antenna Broadcasting with Heterogeneous Delay-Constrained Traffic

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## Abstract

This paper is concerned with dynamic resource allocation in a cellular wireless network with slow fading for support of data traffic having heterogeneous transmission delay requirements. The multiple-input single-output (MISO) fading broadcast channel (BC) is of interest where the base station (BS) employs multiple transmit antennas to realize simultaneous downlink transmission at the same frequency to multiple mobile users each having a single receive antenna. An information-theoretic approach is taken for characterizing capacity limits of the fading MISO-BC under various transmission delay considerations. First, this paper studies transmit optimization at the BS when some users have delay-tolerant “packet” data and the others have delay-sensitive “circuit” data for transmission at the same time. Based on the convex optimization framework, an online resource allocation algorithm is derived that is amenable to efficient cross-layer implementation of both physical (PHY) -layer multi-antenna transmission and media-access-control (MAC) -layer multiuser rate scheduling. Secondly, this paper investigates the fundamental throughput-delay tradeoff for transmission over the fading MISO-BC. By comparing the network throughput under completely relaxed versus strictly zero transmission delay constraint, this paper characterizes the limiting loss in sum capacity due to the vanishing delay tolerance, termed the *delay penalty*, under some prescribed user fairness for transmit rate allocation.

## Index Terms

Broadcast channel (BC), fading channel, multi-antenna, throughput-delay tradeoff, dynamic resource allocation, cross-layer optimization, convex optimization.

## I. INTRODUCTION

In mobile wireless networks, communications typically take place over time-varying channels. When this time-variation or fading is “fast” such that the channel state information (CSI) is hardly obtainable at the transmitter, a classical approach for mitigating impairments of fading to transmission reliability is to apply *diversity techniques*,

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such as coded diversity, antenna diversity, and path diversity. On the other hand, when the fading channel changes sufficiently “slowly” such that the transmitter is able to acquire the CSI, a general approach to compensate for the fading is *dynamic resource allocation*, whereby transmit resources such as power, bit-rate, antenna-beam and bandwidth are dynamically allocated based upon the fading distribution. Effective implementation of dynamic resource allocation usually requires joint optimization of both physical (PHY) -layer transmission and media-access-control (MAC) -layer rate scheduling in the classical communication protocol stack, and thus demands for a new *cross-layer* design methodology.

One challenging issue to be addressed for dynamic resource allocation in tomorrow’s wireless networks is how to meet with user’s heterogeneous transmission quality-of-service (QoS) requirements. Among others, the demand for wireless high-speed connectivity for both delay-tolerant “packet” data and delay-sensitive “circuit” data is expected to rise significantly in the next decade. Therefore, study on both spectral and power efficient transmission schemes for support of *heterogeneous delay-constrained data traffic* becomes an important area for research. On the other hand, because tolerance for a larger transmission delay incurred to data traffic allows for more flexible transmit power and rate adaptation over time and thereby leads to a larger transmission throughput in the long term, there is in general a fundamental *throughput-delay tradeoff* associated with dynamic resource allocation over fading channels. Characterization of such fundamental tradeoff is another important research problem because it reveals the ultimate gain achievable by dynamic resource allocation under realistic transmission delay requirements.

This paper is aimed to provide concrete answers to the aforementioned problems by considering the fading broadcast channel (BC) that models the downlink transmission in a typical wireless cellular network. An information-theoretic approach is taken in this paper to address some fundamental limits of dynamic resource allocation for the fading BC under various transmission delay considerations. In particular, the fading multiple-input single-output (MISO) BC is considered where multi-antennas are equipped at the transmitter of the base station (BS), and single antenna at the receiver of each mobile user. Because of multi-antennas at the transmitter, spatial multiplexing can be used at the BS to support simultaneous transmission to mobile users at the same frequency, named space-division-multiple-access (SDMA). A slow-fading environment is assumed, and for simplicity, the block-fading (BF) channel model is adopted. It is further assumed that the BS has perfect user CSI at its transmitter, and is thus able to perform a centralized dynamic resource allocation based upon multiuser

channel conditions. This paper's main contributions are summarized as follows:

- This paper studies optimal dynamic resource allocation for the fading MISO-BC when both no-delay-constrained (NDC) packet data and delay-constrained (DC) circuit data are required for transmission at the same time. A cross-layer optimization approach is taken for jointly optimizing capacity-achieving multi-antenna transmission at the PHY-layer and fairness-ensured multiuser rate scheduling at the MAC-layer. A convex optimization framework is formulated for minimizing the average transmit power at the BS subject to both NDC and DC user rate constraints. A *two-layer Lagrange-duality method* is shown to be the key for solving this problem. Based on this method, a novel online resource allocation algorithm that is amenable to efficient cross-layer implementation is derived, and its convergence behavior is validated.
- This paper investigates the fundamental throughput-delay tradeoff for the fading MISO-BC under optimal dynamic resource allocation. By taking the difference between the maximum sum-rate of users under NDC and DC transmission subject to the constraint that the rate portion allocated to each user needs to be regulated by the same prescribed *rate-profile*, the paper presents a novel characterization for the limiting loss in sum capacity due to the vanishing delay tolerance, termed the *delay penalty*, for the fading MISO-BC. Thereby, the delay penalty provides the answer to the following interesting question: Comparing no delay constraint versus zero-delay constraint for all users in the network, how much is the maximum percentage of throughput gain achievable for *all* users by optimal dynamic resource allocation?

The capacity region under NDC or DC transmission for a fading single-input single-output (SISO) BC has been characterized in [1], [2], and for a fading SISO multiple-access channel (MAC) in [3], [4]. A similar scenario like in this paper with mixed NDC and DC transmission has also been considered in [5] for the single-user multiple-input multiple-output (MIMO) fading channel, and in [6] for the fading SISO-BC. The comparison of achievable rates between NDC and DC transmission has been considered in [7] for the fading MIMO-BC. However, none of the above prior work has considered transmit optimization with mixed NDC and DC data traffic for the fading MISO-BC, which is addressed in this paper. On the other hand, throughput-delay and power-delay tradeoffs for communications over fading channels by exploiting the combined CSI and data buffer occupancy at the transmitter have been intensively studied in the literature for both single-user and multiuser transmission (e.g., [8] and references therein). In contrast to prior work, this paper studies the throughput-delay tradeoff from a new perspective by characterizing the fundamental delay penalty in the network throughput owing

to stringent (zero) transmission delay constraint imposed by all users. The concept of rate-profile, or its equivalent definitions for specifying some certain fairness in user rate allocation have also been considered for the SISO multiuser channel in [9], [10], and for the MIMO multiuser channel in [11], [12]. However, to the author's best knowledge, application of rate-profile for characterizing the delay penalty in a multiuser fading channel is a novelty of this paper. There has been recently a great deal of study on the real-time resource allocation algorithm, named proportional fair scheduling (PFS) (e.g., [13]-[16] and references therein), which maximizes the network throughput by exploiting the multiuser channel variation and at the same time, maintains certain fairness among users in rate allocation. However, PFS is unable to guarantee any prescribed user rate demand. In this paper, a novel online scheduling algorithm is proposed to ensure that all NDC and DC user rate demands are satisfied with the minimum transmit power consumption at the BS.

The remainder of this paper is organized as follows. Section II illustrates the fading MISO-BC model and provides a summary of known information-theoretic results for it. Section III addresses the optimal cross-layer dynamic resource allocation problem for support of simultaneous transmission of heterogeneous delay-constrained traffic. Section IV characterizes the fundamental throughput-delay tradeoff for the fading MISO-BC. Section V provides the simulation results. Finally, Section VI concludes the paper.

*Notation:* This paper uses upper case boldface letters to denote matrices and lower case boldface letters to indicate vectors. For a square matrix  $\mathbf{S}$ ,  $|\mathbf{S}|$  and  $\mathbf{S}^{-1}$  are its determinant and inverse, respectively. For any general matrix  $\mathbf{M}$ ,  $\mathbf{M}^\dagger$  denotes its conjugate transpose.  $\mathbf{I}$  and  $\mathbf{0}$  indicate the identity matrix and the vector with all zero elements, respectively.  $\|\mathbf{x}\|$  denotes the Euclidean norm of a vector  $\mathbf{x}$ .  $\mathbb{E}_n[\cdot]$  denotes statistical expectation over the random variable  $n$ .  $\mathbb{R}^M$  denotes the  $M$ -dimensional real Euclidean space and  $\mathbb{R}_+^M$  is its non-negative orthant.  $\mathbb{C}^{x \times y}$  is the space of  $x \times y$  matrices with complex number entries. The distribution of a circularly-symmetric complex Gaussian (CSCG) vector with the mean vector  $\mathbf{x}$  and the covariance matrix  $\mathbf{\Sigma}$  is denoted by  $\mathcal{CN}(\mathbf{x}, \mathbf{\Sigma})$ , and  $\sim$  means “distributed as”.  $\{x\}^+$  denotes the non-negative part of a real number  $x$ .

## II. SYSTEM MODEL

A MISO-BC channel with  $K$  mobile users each having a single antenna and a fixed BS having  $M$  antennas is considered, as shown in Fig. 1. Because of multi-antennas at the transmitter, the BS is able to employ SDMA to transmit to multiple users simultaneously at the same bandwidth. It is assumed that the transmission to all users is synchronously divided into consecutive blocks, and the fading occurs from block to block but remains

static within a block of symbols, i.e., a block-fading (BF) model. Furthermore, it is assumed that the fading process is stationary and ergodic. Let  $n$  be the random variable representing the fading state. At fading state  $n$ , the MISO-BC can be considered as a discrete-time channel represented by

$$\begin{bmatrix} y_1(n) \\ \vdots \\ y_K(n) \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1(n) \\ \vdots \\ \mathbf{h}_K(n) \end{bmatrix} \mathbf{x}(n) + \begin{bmatrix} z_1(n) \\ \vdots \\ z_K(n) \end{bmatrix}, \quad (1)$$

where  $y_k(n)$ ,  $\mathbf{h}_k(n)$  and  $z_k(n)$  denote the received signal, the  $1 \times M$  downlink channel vector, and the receiver noise for user  $k$ , respectively, and  $\mathbf{x}(n) \in \mathbb{C}^{M \times 1}$  denotes the transmitted signal vector from the BS. It is assumed that  $z_k(n) \sim \mathcal{CN}(0, 1), \forall n, k$ . The transmitted signal  $\mathbf{x}(n)$  can be further expressed as

$$\mathbf{x}(n) = \sum_{k=1}^K \mathbf{b}_k(n) s_k(n), \quad (2)$$

where  $\mathbf{b}_k(n) \in \mathbb{C}^{M \times 1}$  and  $s_k(n)$  represent the precoding vector and the transmitted codeword symbol for user  $k$ , respectively, at fading state  $n$ . It is assumed that each user employs the optimal Gaussian code-book with normalized codeword symbols, i.e.,  $s_k(n) \sim \mathcal{CN}(0, 1), \forall k, n$ , and the rate of code-book for user  $k$  at fading state  $n$  is denoted as  $r_k(n)$ . The allocated transmit power to user  $k$  at fading state  $n$  is denoted by  $p_k(n)$ , and it can be easily verified that  $p_k(n) = \|\mathbf{b}_k(n)\|^2$ . The total transmit power from the BS at fading state  $n$  is then expressed as  $p(n) = \sum_{k=1}^K p_k(n)$ . Assuming full knowledge of the fading distribution, the BS is able to adapt the transmission power  $p_k(n)$  and rate  $r_k(n)$  (could be both zero for some fading state  $n$ ) allocated to user  $k$  in order to exploit multiuser channel variations over time. Assuming a long-term power constraint (LTPC)  $p^*$  over different fading states, the average transmit power at the BS needs to satisfy  $\mathbb{E}_n[p(n)] \leq p^*$ .

Supposing that  $p(n)$  is given, the achievable rates  $\{r_k(n)\}$  of users need to be contained in the corresponding capacity region of the MISO-BC at fading state  $n$ , denoted by  $\mathcal{C}_n^{\text{BC}}(p(n), \{\mathbf{h}_k(n)\})$ . Characterization of  $\mathcal{C}_n^{\text{BC}}$  will become useful later in this paper when the issue on how to dynamically allocate transmit power and user rates at different fading states is addressed. In many cases, it is more convenient to apply the celebrated duality result between the Gaussian BC and MAC [17] to transform the capacity region characterization for the original BC to that for its dual MAC. Assuming that in the dual SIMO-MAC of the original MISO-BC considered in this paper, each user employs the optimal Gaussian code-book of rate  $R_k(n)$  and has a transmit power  $q_k(n)$ ,  $k = 1, \dots, K$ , at fading state  $n$ , by [18] the capacity region of the dual SIMO-MAC at fading state  $n$  can be expressed as

$$\mathcal{C}_n^{\text{MAC}}(\{q_k(n)\}, \{\mathbf{h}_k^\dagger(n)\}) = \left\{ \mathbf{R}(n) \in \mathbb{R}_+^K : \sum_{k \in J} R_k(n) \leq \log_2 \left| \sum_{k \in J} \mathbf{h}_k^\dagger(n) \mathbf{h}_k(n) q_k(n) + \mathbf{I} \right|, \forall J \subseteq \{1, \dots, K\} \right\}, \quad (3)$$

where  $\mathbf{R}(n) \triangleq [R_1(n), \dots, R_K(n)]$ . The duality result [17] then states that an achievable rate region for the original MISO-BC at fading state  $n$  with total transmit power  $p(n)$  can be expressed as

$$\mathcal{R}_n^{\text{BC}}(p(n), \{\mathbf{h}_k(n)\}) = \bigcup_{\{q_k(n) : q_1(n) + \dots + q_K(n) \leq p(n)\}} \mathcal{C}_n^{\text{MAC}}(\{q_k(n)\}, \{\mathbf{h}_k^\dagger(n)\}). \quad (4)$$

It was later shown in [19] that the above achievable rate region  $\mathcal{R}_n^{\text{BC}}(p(n), \{\mathbf{h}_k(n)\})$  is indeed the capacity region  $\mathcal{C}_n^{\text{BC}}(p(n), \{\mathbf{h}_k(n)\})$  for the Gaussian BC. By applying the above results, it follows that any rate-tuple  $\{r_k(n)\}$  that is achievable in the fading MISO-BC by the user power-tuple  $\{p_k(n)\}$  is also achievable as  $\{R_k(n)\}$ ,  $R_k(n) = r_k(n), \forall k, n$ , in the dual fading SIMO-MAC by the corresponding user power-tuple  $\{q_k(n)\}$  provided that  $\sum_{k=1}^K p_k(n) = \sum_{k=1}^K q_k(n), \forall n$ . Note that the power allocation  $p_k(n)$  for user  $k$  in the original BC is not necessarily equal to  $q_k(n)$  in the dual MAC. The transforms between  $\{q_k(n)\}$  and  $\{p_k(n)\}$  as well as the corresponding precoding vectors  $\{\mathbf{b}_k(n)\}$  for the same set of achievable rates  $\{r_k(n)\}$  and  $\{R_k(n)\}$  can be found in [17], and are thus omitted in this paper for brevity.

### III. DYNAMIC RESOURCE ALLOCATION UNDER HETEROGENEOUS DELAY CONSTRAINTS

This section studies optimal dynamic resource allocation algorithms for the BF MISO-BC to support simultaneous transmission of data traffic with heterogeneous transmit rate and delay constraints. First, Section III-A provides the problem formulation. Then, Section III-B presents the solution based on the Lagrange-duality method of convex optimization. At last, Section III-C derives an online algorithm that is suitable for real-time implementation of the proposed solution.

#### A. Problem Formulation

The following rule for transmission scheduling at the BS is considered. As illustrated in Fig. 2, each user's data arising from some higher layer application is first placed into a dedicated buffer. Periodically, the BS removes some of the data from each user's buffer, jointly encodes them into a block of symbols, and then broadcasts the encoded block to all users through the MISO-BC. For simplicity, it is assumed that all user's data arrive to their dedicated buffers synchronously at the beginning of each scheduling period. The data arrival processes of users are assumed to be stationary and ergodic, mutually independent, and also independent of their channel realizations. This paper considers two types of data traffic with very different delay requirements: One is the

delay-tolerant packet data and the other is the delay-sensitive circuit data, for which the following assumptions are made:

- For a user with packet data application, the data arrival process is not necessarily continuous in time and the amount of arrived data in each scheduling period may be variable. All data are stored in a buffer of a sufficiently large size such that data dropping due to buffer overflow does not occur. In order for the scheduler to optimally exploit the channel dynamics, the allocated transmit rate can be variable during each scheduling period. It is assumed that there is always a sufficient amount of backlogged data in the buffer for transmission. The scheduler needs to ensure that the transmit rate averaged over scheduling periods in the long run is no smaller than the average data arrival rate. However, the exact amount of delay incurred to transmitted data in the buffer is not guaranteed.
- For a user with circuit data application, the data arrival process is continuous with a constant-rate during each scheduling period. The arrived data is stored in the buffer for only one scheduling period and then transmitted. Therefore, the amount of delay incurred to transmitted data is minimal. However, the scheduler needs to ensure a constant-rate transmission independent of channel condition.

Let the users with packet data applications be represented by the set  $\mathcal{U}_{\text{NDC}}$  where NDC refers to no-delay-constrained, and the users with circuit data applications represented by  $\mathcal{U}_{\text{DC}}$  where DC refers to delay-constrained. In this paper, we consider optimal dynamic resource allocation to minimize the average transmit power at the BS over different fading states subject to the constraint that all NDC and DC user rate demands are satisfied. Recall that  $r_k(n)$  denotes the rate assigned to user  $k$  by the scheduler at fading state  $n$ . For a NDC user, it is required that the average transmit rate  $\mathbb{E}_n[r_k(n)]$  over fading states needs to be no smaller than its average data arrival rate  $R_k^*$ . In contrast, for a DC user, the transmit rate  $r_k(n)$  at any fading state  $n$  needs to satisfy its constant data arrival rate  $R_k^*$ . By considering the dual SIMO-MAC in Section II with  $R_k(n) = r_k(n)$ ,  $q_k(n) = p_k(n)$ ,  $\forall k, n$ , optimal allocation of transmit rates and powers at different  $n$  can be obtained by solving the following optimization

problem **(P1)**:

$$\text{Minimize} \quad \mathbb{E}_n \left[ \sum_{k=1}^K q_k(n) \right] \quad (5)$$

$$\text{Subject to} \quad \mathbb{E}_n[R_k(n)] \geq R_k^*, \quad \forall k \in \mathcal{U}_{\text{NDC}} \quad (6)$$

$$R_k(n) \geq R_k^*, \quad \forall n, \quad \forall k \in \mathcal{U}_{\text{DC}} \quad (7)$$

$$\mathbf{R}(n) \in \mathcal{C}_n^{\text{MAC}} \left( \{q_k(n)\}, \{\mathbf{h}_k^\dagger(n)\} \right), \quad \forall n \quad (8)$$

$$q_k(n) \geq 0 \quad \forall n, k. \quad (9)$$

For the above problem, the objective function and all the constraints except (8) are affine. It can also be verified from (3) that  $\mathcal{C}_n^{\text{MAC}}$  is a convex set with any given positive  $\{q_k(n)\}$ . Therefore, Problem P1 is a convex optimization problem [20] and thus can be solved using efficient convex optimization techniques, as will be shown next.

### B. Proposed Solution

The Lagrange-duality method is usually applied when a convex optimization problem can be more conveniently solved in its dual domain than in its original form. In this paper, we also apply this method for solving Problem P1. The first step for the Lagrange-duality method is to introduce dual variables associated with some constraints of the original problem. For Problem P1 that has multiple constraints, there are also various ways to introduce dual variables that might result in different dual problems. For the following proposed solution, dual variables are chosen with an aim to facilitate implementing it in the real time, as will be explained later in Section III-C.

As a first step, a set of dual variables  $\{\mu_k\}$ ,  $\mu_k \geq 0$ ,  $k \in \mathcal{U}_{\text{NDC}}$ , are introduced for the NDC users with respect to (w.r.t.) their average-rate constraints in (6). The Lagrangian of Problem P1 can be then expressed as

$$\mathcal{L}(\{q_k(n)\}, \{R_k(n)\}, \{\mu_k\}) = \mathbb{E}_n \left[ \sum_{k=1}^K q_k(n) \right] - \sum_{k \in \mathcal{U}_{\text{NDC}}} \mu_k (\mathbb{E}_n[R_k(n)] - R_k^*). \quad (10)$$

Denote the set of  $\{q_k(n)\}$  and  $\{R_k(n)\}$  specified by the remaining constraints in (7), (8) and (9) as  $\mathcal{D}$ , the Lagrange dual function is expressed as

$$g(\{\mu_k\}) = \min_{\{q_k(n), R_k(n)\} \in \mathcal{D}} \mathcal{L}(\{q_k(n)\}, \{R_k(n)\}, \{\mu_k\}). \quad (11)$$

The dual problem of the original (primal) problem can be then expressed as

$$\max_{\mu_k \geq 0, k \in \mathcal{U}_{\text{NDC}}} g(\{\mu_k\}). \quad (12)$$



Because the primal problem is convex and also satisfies the Slater's condition [20],<sup>1</sup> the duality gap between the optimal value of the primal problem and that of the dual problem becomes zero. This suggests that the problem at hand can be equivalently solved in its dual domain by first minimizing the Lagrangian  $\mathcal{L}$  to obtain the dual function  $g(\{\mu_k\})$  for some given  $\{\mu_k\}$ , and then maximizing  $g(\{\mu_k\})$  over  $\{\mu_k\}$ .

Considering first the minimization problem in (11) to obtain  $g(\{\mu_k\})$  for some given  $\{\mu_k\}$ . It is interesting to observe that this problem can be solved by considering parallel subproblems each corresponding to one fading state  $n$ . From (10), the subproblem for fading state  $n$  can be written as **(P2)**

$$\text{Minimize} \quad \sum_{k=1}^K q_k(n) - \sum_{k \in \mathcal{U}_{\text{NDC}}} \mu_k R_k(n) \quad (13)$$

$$\text{Subject to} \quad R_k(n) \geq R_k^*, \quad \forall k \in \mathcal{U}_{\text{DC}} \quad (14)$$

$$\mathbf{R}(n) \in \mathcal{C}_n^{\text{MAC}}(\{q_k(n)\}, \{\mathbf{h}_k^\dagger(n)\}) \quad (15)$$

$$q_k(n) \geq 0 \quad \forall k. \quad (16)$$

Hence, the dual function  $g(\{\mu_k\})$  can be obtained by solving subproblems all having the identical structure, a technique usually referred to as the *Lagrange-dual decomposition*. For solving Problem P2 for each  $n$ , a new set of positive dual variables  $\delta_k(n), k \in \mathcal{U}_{\text{DC}}$ , are introduced for DC users w.r.t. their constant-rate constraints in (14). The Lagrangian of Problem P2 can be then expressed as

$$\mathcal{L}_n(\{q_k\}, \{R_k\}, \{\delta_k\}) = \sum_{k=1}^K q_k - \sum_{k \in \mathcal{U}_{\text{NDC}}} \mu_k R_k - \sum_{k \in \mathcal{U}_{\text{DC}}} \delta_k (R_k - R_k^*). \quad (17)$$

Note that for brevity, the index  $n$  is dropped in  $q_k(n)$ ,  $R_k(n)$  and  $\delta_k(n)$  in (17) since it is applicable for all  $n$ .

The corresponding dual function can be then defined as

$$g_n(\{\delta_k\}) = \min_{\{q_k, R_k\} \in \mathcal{D}_n} \mathcal{L}_n(\{q_k\}, \{R_k\}, \{\delta_k\}), \quad (18)$$

where  $\mathcal{D}_n$  denotes the set of  $\{q_k\}$  and  $\{R_k\}$  specified by the remaining constraints (15) and (16) at fading state  $n$ . The associated dual problem is then defined as

$$\max_{\delta_k \geq 0, k \in \mathcal{U}_{\text{DC}}} g_n(\{\delta_k\}). \quad (19)$$

Similar like P1, Problem P2 can also be solved by first minimizing  $\mathcal{L}_n$  to obtain the dual function  $g_n(\{\delta_k\})$  for a given set of  $\{\delta_k\}$ , and then maximizing  $g_n$  over  $\{\delta_k\}$ . From (17), the minimization problem in (18) can be

<sup>1</sup>Slater's condition requires that the feasible set of the optimization problem has non-empty interior, which is in general the case for Problem P1 because with sufficiently large average transmit power, any finite user rates  $\{R_k^*\}$  are achievable.

expressed as

$$\text{Minimize} \quad \sum_{k=1}^K q_k - \sum_{k \in \mathcal{U}_{\text{NDC}}} \mu_k R_k - \sum_{k \in \mathcal{U}_{\text{DC}}} \delta_k R_k \quad (20)$$

$$\text{Subject to} \quad \mathbf{R} \in \mathcal{C}_n^{\text{MAC}} \left( \{q_k\}, \{\mathbf{h}_k^\dagger\} \right) \quad (21)$$

$$q_k \geq 0 \quad \forall k. \quad (22)$$

Let  $\beta_k = \mu_k$ , if  $k \in \mathcal{U}_{\text{NDC}}$ , and  $\beta_k = \delta_k$ , if  $k \in \mathcal{U}_{\text{DC}}$ , and  $\pi$  be a permutation over  $\{1, \dots, K\}$  such that  $\beta_{\pi(1)} \geq \beta_{\pi(2)} \geq \dots \geq \beta_{\pi(K)}$ , and let  $\beta_{\pi(K+1)} \triangleq 0$ . Thanks to the polymatroid structure of  $\mathcal{C}_n^{\text{MAC}}$  [3], the above problem can be simplified as **(P3)**

$$\text{Minimize} \quad \sum_{k=1}^K q_k - \sum_{k=1}^K (\beta_{\pi(k)} - \beta_{\pi(k+1)}) \log_2 \left| \sum_{i=1}^k \mathbf{h}_{\pi(i)}^\dagger \mathbf{h}_{\pi(i)} q_{\pi(i)} + \mathbf{I} \right| \quad (23)$$

$$\text{Subject to} \quad q_k \geq 0 \quad \forall k. \quad (24)$$

Problem P3 is convex because all the constraints are affine and the objective function is convex w.r.t.  $\{q_k\}$ . Hence, this problem can be solved, e.g., by the interior-point method [20]. In Appendix I, an alternative method based on the block-coordinate decent principle [21] by iteratively optimizing  $q_k$  with all the other  $\{q_{k'}\}, k' \neq k$  as fixed is presented. This method can be considered as a generalization of the algorithm described in [22], where all  $\beta_{\pi(k)}, k = 1, \dots, K$ , are equal.

So far, solutions have been presented for the minimization problem in (11) to obtain  $g(\{\mu_k\})$  for some given  $\{\mu_k\}$  and that in (18) to obtain each  $g_n(\{\delta_k\})$  for some given  $\{\delta_k\}$ . Next, the remaining issue on how to update  $\{\mu_k\}$  to maximize  $g(\{\mu_k\})$  for the dual problem in (12) is addressed. Similar techniques can also be used for updating  $\{\delta_k\}$  to maximize  $g_n(\{\delta_k\})$  for each dual problem in (19). From (10) and (11), it is observed that the dual function  $g(\{\mu_k\})$ , though affine w.r.t.  $\{\mu_k\}$ , is not directly differentiable w.r.t.  $\{\mu_k\}$ . Hence, standard method like Newton method cannot be employed to update  $\{\mu_k\}$  for maximizing  $g(\{\mu_k\})$ . An appropriate choice here may be the *sub-gradient-based* method [21] that is capable of handling non-differentiable functions. This method is an iterative algorithm and at each iteration, it requires a sub-gradient at the corresponding value of  $\{\mu_k\}$  to update  $\{\mu_k\}$  for the next iteration. Suppose that after solving Problem P2 for some given  $\{\mu_k\}$  at all fading states of  $n$ , the obtained rates and powers are denoted by  $\{R'_k(n)\}$  and  $\{q'_k(n)\}$ , respectively,  $k \in \mathcal{U}_{\text{NDC}}$ . The following lemma then provides a suitable sub-gradient for  $\{\mu_k\}$ :

*Lemma 3.1:* If  $\mathcal{L}(\{q'_k(n)\}, \{R'_k(n)\}, \{\mu_k\}) = g(\{\mu_k\})$ , then the vector  $\boldsymbol{\nu}$  defined as  $\nu_k = R_k^* - \mathbb{E}_n[R'_k(n)]$  for  $k \in \mathcal{U}_{\text{NDC}}$  is a sub-gradient of  $g$  at  $\{\mu_k\}$ .

*Proof:* Since for any set of  $\theta_k$ 's,  $\theta_k \geq 0, k \in \mathcal{U}_{\text{NDC}}$ , it follows that

$$g(\{\theta_k\}) \leq \mathcal{L}(\{q'_k(n)\}, \{R'_k(n)\}, \{\theta_k\}) \quad (25)$$

$$= g(\{\mu_k\}) + \sum_{k \in \mathcal{U}_{\text{NDC}}} (\theta_k - \mu_k) (R_k^* - \mathbb{E}_n[R'_k(n)]). \quad (26)$$

Hence, it is clear that the optimal dual solution  $\{\mu_k^*\}$  that maximizes  $g(\{\mu_k\})$  should not lie in the half space represented by  $\{\theta : \sum_{k \in \mathcal{U}_{\text{NDC}}} (\theta_k - \mu_k) \nu_k \leq 0\}$ . As a result, any new update of  $\{\mu_k\}$  for the next iteration, denoted by  $\{\mu_k^{\text{new}}\}$ , should satisfy  $\sum_{k \in \mathcal{U}_{\text{NDC}}} (\mu_k^{\text{new}} - \mu_k) \nu_k \geq 0$ . ■

By applying Lemma 3.1, a simple rule for updating  $\{\mu_k\}$  is as follows:<sup>2</sup>

$$\mu_k^{\text{new}} = \{\mu_k + \Delta (R_k^* - \mathbb{E}_n[R'_k(n)])\}^+, \quad k \in \mathcal{U}_{\text{NDC}}, \quad (27)$$

where  $\Delta$  is a small positive step size. Similarly, at each fading state  $n$ ,  $\{\delta_k(n)\}$  can be also updated towards maximizing  $g_n(\{\delta_k(n)\})$  as

$$\delta_k^{\text{new}}(n) = \{\delta_k(n) + \Delta (R_k^* - R'_k(n))\}^+, \quad k \in \mathcal{U}_{\text{DC}}, \quad (28)$$

where  $\{R'_k(n)\}$ ,  $k \in \mathcal{U}_{\text{DC}}$ , is the solution obtained after solving Problem P3 at fading state  $n$ .

To summarize, the proposed solution is implemented by a *two-layer* Lagrange-duality method. At the first layer, the algorithm searches iteratively for  $\{\mu_k^*\}$  with which the average-rate constraints of all NDC users are satisfied. At each iteration, an update for  $\{\mu_k\}$  is generated and then passed to the second layer where the algorithm starts a parallel search for  $\{\delta_k^*(n)\}$ , each for a fading state  $n$ , such that the constant-rate constraints of all DC users are satisfied at all fading states. The resultant  $\{R'_k(n)\}$  of NDC users is then passed back to the first layer for another update of  $\{\mu_k\}$ . The overall algorithm is summarized in Table I. The complexity of this algorithm can be derived as follows. Supposing that the ellipsoid method is used to iteratively update dual variables, the required number of iterations for convergence has  $\mathcal{O}(m^2)$ , where  $m$  is the size of the problem. Let  $K_{\text{NDC}}$  and  $K_{\text{DC}}$  denote the size of  $\mathcal{U}_{\text{NDC}}$  and  $\mathcal{U}_{\text{DC}}$ , respectively, where  $K_{\text{NDC}} + K_{\text{DC}} = K$ . Therefore, the ellipsoid method will need  $\mathcal{O}(K_{\text{NDC}}^2)$  iterations for obtaining  $\{\mu_k^*\}$  and  $\mathcal{O}(NK_{\text{DC}}^2)$  iterations for obtaining  $\{\delta_k^*(n)\}$  for all fading states, assuming the number of fading states  $n$  is finite and is equal to  $N$ . The complexity for solving Problem P3 is  $\mathcal{O}(K^2)$  by, e.g., the interior-point method. Hence, the total complexity of the algorithm is  $\mathcal{O}(K_{\text{NDC}}^2 K_{\text{DC}}^2 K^2 N)$ .

<sup>2</sup>Notice that a more efficient sub-gradient-based method to iteratively find  $\{\mu_k^*\}$  is the ellipsoid method [23], which at each iteration removes the half space specified by  $\{\theta : \sum_{k \in \mathcal{U}_{\text{NDC}}} (\theta_k - \mu_k) \nu_k \leq 0\}$  for searching  $\{\mu_k^*\}$ .

At last, take note that the proposed algorithm jointly optimizes transmit powers and rates together with decoding orders (determined by the magnitudes of  $\{\mu_k^*\}$  and  $\{\delta_k^*(n)\}$  [3]) of users at all fading states for the dual fading SIMO-MAC. By the BC-MAC duality result [17], the optimal transmit powers, rates, precoding vectors as well as encoding orders of users for the original fading MISO-BC can be obtained.

### C. Online Algorithm

One important issue yet to be addressed for implementing the proposed solution for Problem P1 is how to relax its assumption on perfect knowledge of the distribution of fading state  $n$ . Notice that this knowledge is necessary for computing the achievable average rates  $\{\mathbb{E}_n[R'_k(n)]\}$ ,  $k \in \mathcal{U}_{\text{NDC}}$ , which are needed for updating the dual variables  $\{\mu_k\}$  for NDC users in (27). In practice, although it is reasonable to assume each user's fading channel is stationary and ergodic, the space of fading states is usually continuous and infinite and hence it is infeasible for the BS to initially acquire the channel distribution information for all users at all fading states. Even though this information is available for off-line implementation of the proposed solution, the computational complexity becomes unbounded as the number of fading states goes to infinity. Therefore, in this paper a modified "online" algorithm is developed that is able to adaptively update  $\{\mu_k\}$  towards  $\{\mu_k^*\}$  as transmission proceeds over time. Let  $t$  denote the transmission block index,  $t = 1, 2, \dots$ . The key for the online algorithm is to approximate the statistical average  $\mathbb{E}_n[R'_k(n)]$ ,  $k \in \mathcal{U}_{\text{NDC}}$ , at time  $t$  by a time average of transmitted rates up to time  $t - 1$ , denoted by  $\bar{R}_k[t - 1]$ , where  $\bar{R}_k[t]$  is obtained as

$$\bar{R}_k[t] = (1 - \epsilon)\bar{R}_k[t - 1] + \epsilon R_k[t], \quad (29)$$

where  $R_k[t]$  is the transmitted rate at time  $t$ , and  $\epsilon$ ,  $0 < \epsilon \ll 1$ , is a parameter that controls the convergence speed of  $\bar{R}_k[t] \rightarrow \mathbb{E}_n[R'_k(n)]$  as  $t \rightarrow \infty$ . By replacing  $\mathbb{E}_n[R'_k(n)]$  by  $\bar{R}_k[t - 1]$  in (27),  $\{\mu_k[t]\}$  at time  $t$  can be updated accordingly such that as  $t \rightarrow \infty$  it converges to  $\{\mu_k^*\}$  because  $\bar{R}_k[t] \rightarrow R_k^*$ ,  $\forall k \in \mathcal{U}_{\text{NDC}}$ . This modified online algorithm is summarized in Table II.

**Cross-Layer Implementation:** The proposed online algorithm based on the Lagrange-duality method is amenable to cross-layer implementation of both PHY-layer transmission and MAC-layer multiuser rate scheduling. One challenging issue for cross-layer optimization is on how to select useful and succinct information for different layers to exchange and share so as to optimize their individual operations. The Lagrange-duality method provides a new design paradigm for efficient cross-layer information exchange. On the one hand, since the MAC-layer has

the knowledge of user rate demands  $\{R_k^*\}$  as well as their delay requirements (NDC or DC), it can update dual variables  $\{\mu_k[t]\}$  and  $\{\delta_k[t]\}$  accordingly (see Table II), and then pass them to the PHY-layer for computing the desirable transmission rates of users  $\{R_k[t]\}$  to meet with each user's specific rate demand. On the other hand, the PHY-layer is able to provide the MAC-layer the updated  $\{R_k[t]\}$  that optimize the PHY-layer transmission by solving Problem P3 given  $\{\mu_k[t]\}$  and  $\{\delta_k[t]\}$ . Therefore, by exchanging dual variables and transmission rates between PHY- and MAC-layers, cross-layer dynamic resource allocation can be efficiently implemented.

**Comparison with PFS:** It is interesting to draw a comparison between the proposed online algorithm and the well-known PFS algorithm. PFS is designed for real-time multiuser rate scheduling in a mobile wireless network to ensure some certain fairness for user rate allocation while maximizing the network throughput. PFS applies to packet data transmission and hence is equivalent to the transmission scenario considered in this paper when only NDC users are present. At each time  $t$ , transmit rates  $\{R_k[t]\}$  assigned to users by PFS maximize the weighted sum-rate of users  $\sum_k \omega_k[t] R_k[t]$ , where the weights are given by  $\omega_k[t] = \frac{1}{R_k[t-1]}$  and  $\bar{R}_k[t-1]$  is the estimated average rate for user  $k$  up to time  $t-1$  the same as expressed in (29). Using this rule, it has been shown (e.g., [13]-[16] and references therein) that as  $t \rightarrow \infty$ ,  $\bar{R}_k[t] \rightarrow R_k^*$  where  $R_k^*$  is the average achievable rate for user  $k$  in the long term. Furthermore, PFS maximizes  $\sum_k \log(R_k^*)$  over the expected capacity region (please refer to Definition 4.1 in Section IV) and, hence,  $\{R_k^*\}$  can be considered as the unique intersection of the surface specified by  $\prod_k R_k = c$  and the boundary of the expected capacity region. Because of the  $\log(\cdot)$  function, the rate assignments among users by PFS are regulated in a balanced manner such that no user can be allocated an overwhelmingly larger rate than the others even if it has a superior channel condition. However, the achievable rates  $\{R_k^*\}$  by PFS are not guaranteed to satisfy any desired rate demand of users. In contrast, the proposed online algorithm ensures that each NDC user's average-rate demand is satisfied by applying a different rule (see Table II) for updating user weights  $\{\mu_k[t]\}$  for the resource allocation problem (see Problem P3) to be solved at each  $t$ .

#### IV. THROUGHPUT-DELAY TRADEOFF

For a single-user BF channel, the expected capacity [24], [25] and the delay-limited capacity [4] can be considered as the fading channel capacity limits under two extreme cases of delay constraint. The former is always larger than the latter and their difference, termed the *delay penalty*, then characterizes a fundamental throughput-delay tradeoff for dynamic resource allocation over a single-user fading channel. The delay penalty

may or may not be significantly large for a single-user fading channel. For example, for a SISO-BF channel, the delay penalty can be substantial because the delay-limited capacity is indeed zero if the fading channel is not “invertible” with a finite average transmit power [26]. However, when multi-antennas are employed at the transmitter and/or receiver, the delay-limited capacity of the MIMO-BF channel can be very close to the channel expected capacity [27], i.e., a negligible delay penalty. This result can be explained by the “channel-hardening” effect [28] for random MIMO channels, i.e., the mutual information of independent MIMO channels becomes less variant because of antenna-induced space diversity, and hence the value of power and rate adaptation over time vanishes as the number of antennas becomes large. This result indicates that from an information-theoretic viewpoint, MIMO channels are highly suitable for transmission of real-time and delay-constrained data traffic.

Characterization of the delay penalty in a multiuser fading channel is more challenging. The capacity definitions for the single-user fading channel can be extended to the multiuser channel as the *expected capacity region* under no delay constraint for all users, and the *delay-limited capacity region* under zero-delay constraint for all users. Therefore, the delay penalty can be measured by directly comparing these two capacity regions. Since capacity region contains the achievable rates of all users, it lies in a  $K$ -dimensional space where  $K$  is the number of users in the network. As a result, characterization of capacity region becomes inconvenient as  $K$  becomes large. In order to overcome this difficulty, prior research work usually adopts the maximum sum-rate of users over the capacity region, termed the *sum capacity*, as a simplified measure for the network throughput. Applying the sum capacity to the corresponding capacity region, the *expected throughput* and the *delay-limited throughput* can be defined accordingly. However, the conventional sum capacity does not consider the rate allocation among users. As a consequence, each user’s allocated rate portion in the expected throughput may be different compared to that in the delay-limited throughput. Hence, a similar measure for the delay penalty like in the single-user case by taking the difference between the expected and delay-limited throughput looks problematic at a first glance in the multiuser case.

This section presents a novel characterization of the fundamental throughput-delay tradeoff for the fading MISO-BC. Instead of considering mixed NDC and DC transmission like Section III, it is assumed here that there are only NDC or DC users present, and comparison of the network throughput under these two extreme cases of delay constraint is of interest. Because it is hard to compare directly the expected and the delay-limited capacity region as the number of users becomes large, the sum capacity is also considered for simplicity. However, unlike

the conventional sum capacity that does not guarantee the amount of rate allocation among users, the expected and delay-limited throughput in this paper are defined under a new constraint that regulates each user's allocated rate portion based upon a prescribed *rate-profile*. Then, the delay penalty is characterized by the difference between the expected and delay-limited throughput *under the same rate-profile*. The above concepts are more explicitly defined as follows:

*Definition 4.1:* The expected capacity region for a fading MISO-BC expressed in (1) under a LTPC  $p^*$  can be defined as

$$\mathcal{C}_e(p^*) = \{\mathbf{r} \in \mathbb{R}_+^K : r_k = \mathbb{E}_n[R_k(n)], \forall k, \mathbf{R}(n) \in \mathcal{C}_n^{\text{BC}}(p(n), \{\mathbf{h}_k(n)\}), \forall n, \mathbb{E}_n[p(n)] \leq p^*\}. \quad (30)$$

Similarly, the delay-limited capacity region is defined as

$$\mathcal{C}_d(p^*) = \{\mathbf{r} \in \mathbb{R}_+^K : r_k = R_k(n), \forall k, n, \mathbf{R}(n) \in \mathcal{C}_n^{\text{BC}}(p(n), \{\mathbf{h}_k(n)\}), \forall n, \mathbb{E}_n[p(n)] \leq p^*\}. \quad (31)$$

*Definition 4.2:* Let  $R_k^*$  denote  $k$ -th user's rate demand (average-rate for a NDC user or constant-rate for a DC user)  $k = 1, \dots, K$ , the rate profile is defined as a vector  $\boldsymbol{\alpha} = \{\alpha_1, \dots, \alpha_K\}$ , where  $\alpha_k = \frac{R_k^*}{\sum_{k=1}^K R_k^*}, k = 1, \dots, K$ .

*Definition 4.3:* The expected throughput  $C_e(p^*, \boldsymbol{\alpha})$  (delay-limited throughput  $C_d(p^*, \boldsymbol{\alpha})$ ) associated with a prescribed rate-profile  $\boldsymbol{\alpha}$  under a LTPC  $p^*$  is defined as the maximum sum-rate of users over the expected (delay-limited) capacity region under the constraint that the average (constant) transmit rate of each user  $r_k^*$  must satisfy  $\frac{r_i^*}{r_j^*} = \frac{\alpha_i}{\alpha_j}, \forall i, j \in \{1, \dots, K\}$ .

*Definition 4.4:* For some given delay profile  $\boldsymbol{\alpha}$  and LTPC  $p^*$ , the delay penalty  $C_{\text{DP}}(p^*, \boldsymbol{\alpha})$  is equal to  $C_e(p^*, \boldsymbol{\alpha}) - C_d(p^*, \boldsymbol{\alpha})$ .

The proposed definition for delay penalty is illustrated in Fig. 3 for a 2-user case. From Definition 4.4, it is noted that characterization of  $C_{\text{DP}}(p^*, \boldsymbol{\alpha})$  requires that of both expected and delay-limited throughput. In the next, we present the algorithm for characterizing  $C_e(p^*, \boldsymbol{\alpha})$  for some given  $p^*$  and  $\boldsymbol{\alpha}$ . Similar algorithm can also be developed for characterizing  $C_d(p^*, \boldsymbol{\alpha})$  and is thus omitted here for brevity. According to Definition 4.3 and (30) and using the BC-MAC duality result in Section II, the expected throughput  $C_e(p^*, \boldsymbol{\alpha})$  can be obtained by

considering the following optimization problem **(P4)**:

$$\text{Maximize} \quad C_e \quad (32)$$

$$\text{Subject to} \quad \mathbb{E}_n[R_k(n)] \geq C_e \alpha_k, \quad (33)$$

$$\mathbf{R}(n) \in \mathcal{C}_n^{\text{MAC}} \left( \{q_k(n)\}, \{\mathbf{h}_k^\dagger(n)\} \right), \quad \forall n \quad (34)$$

$$q_k(n) \geq 0 \quad \forall n, k. \quad (35)$$

$$\mathbb{E}_n \left[ \sum_{k=1}^K q_k(n) \right] \leq p^* \quad (36)$$

Similar like Problem P1, it can be verified that the above problem is also convex, and hence can be solved using convex optimization techniques. Here, instead of solving Problem P4 from a scratch, the proposed solution transforms this problem into a special form of Problem P1 and hence the same algorithm for Problem P1 can be applied. First, considering the following transmit power minimization problem **(P5)** for support of any arbitrary set of average-rate demands  $\{R_k^*\}$  that satisfy a given rate-profile constraint  $\alpha$ , i.e.,  $\frac{R_i^*}{R_j^*} = \frac{\alpha_i}{\alpha_j}$ ,  $\forall i, j \in \{1, \dots, K\}$ :

$$\text{Minimize} \quad \mathbb{E}_n \left[ \sum_{k=1}^K q_k(n) \right] \quad (37)$$

$$\text{Subject to} \quad \mathbb{E}_n[R_k(n)] \geq R_{\text{sum}}^* \alpha_k, \quad \forall k, \quad (38)$$

$$\mathbf{R}(n) \in \mathcal{C}_n^{\text{MAC}} \left( \{q_k(n)\}, \{\mathbf{h}_k^\dagger(n)\} \right), \quad \forall n \quad (39)$$

$$q_k(n) \geq 0 \quad \forall n, k. \quad (40)$$

Note that  $R_{\text{sum}}^* \triangleq \sum_{k=1}^K R_k^*$ . It is observed that the above problem is a special case of Problem P1 if all users have NDC transmission, i.e., those constant-rate constraints for DC users in (7) are removed. Hence, Problem P5 can also be solved by the proposed algorithm in Table I. Let  $q^*$  denote the minimal transmit power obtained after solving Problem P5. Notice that  $C_e(p^*, \alpha)$  is a non-decreasing function of  $p^*$  with some given  $\alpha$  because the expected capacity region  $\mathcal{C}_e(p^*)$  corresponding to a larger  $p^*$  always contains that with a smaller  $p^*$ . Hence, if  $p^* > q^*$ , it can be inferred that  $C_e(p^*, \alpha)$  must be larger than the assumed  $R_{\text{sum}}^*$ . Otherwise,  $C_e(p^*, \alpha) \leq R_{\text{sum}}^*$ . By using this property,  $C_e(p^*, \alpha)$  can be easily obtained by a bisection search [20].

**Fairness Penalty:** There is an interesting relationship between the conventional sum capacity over the expected capacity region, and the expected throughput as a function of delay-profile, as shown in Fig. 4 for a 2-user case. The sum capacity is obtained by maximizing the sum-rate of users over the expected capacity region so as to maximally exploit the multiuser diversity gain in the achievable network throughput. However, it does not



guarantee the resultant rate allocation among users. Let the resultant rate portion allocated to each user in the sum capacity be specified by a rate-profile vector  $\alpha^*$ . In contrast, the expected throughput maximizes the sum-rate of users under any arbitrary rate-profile vector  $\alpha_e$ . Due to this hard fairness constraint, the expected throughput in general is smaller than the sum capacity if  $\alpha_e$  is different from  $\alpha^*$ , and their difference can be used as a measure of the *fairness penalty*, which is denoted by  $\mathcal{C}_{\text{FP}}(p^*, \alpha_e) \triangleq C_e(p^*, \alpha^*) - C_e(p^*, \alpha_e)$ . Similarly, the fairness penalty can also be defined in the delay-limited case.

**Asymptotic Results:** Consider the MISO-BC with asymptotically large number of users  $K$  but fixed number of transmit antennas  $M$  at the BS. It is assumed that the network is homogeneous where all users have mutually independent but identically distributed channels, and have identical rate demands, i.e.,  $\alpha_k = \frac{1}{K}, \forall k$ . Under these assumptions, in [29] it has been shown that as  $K \rightarrow \infty$  the expected throughput  $C_e$  under any finite power constraint  $p^*$  scales like  $M \log_2 \log K + \mathcal{O}(1)$ . In the following theorem, we provide this asymptotic result for the delay-limited throughput:

*Theorem 4.1:* Under the assumption of symmetric fading and symmetric user rate demand, the delay-limited throughput  $C_d$  for a fading MISO-BC under a LTPC  $p^*$  is upper-bounded by  $\frac{p^*}{\rho \log 2}$  as  $K \rightarrow \infty$ , where  $\rho$  is a constant depending solely on the channel distribution.

*Proof:* Please refer to Appendix II. ■

The above results suggest very different behaviors of the achievable network throughput with NDC versus DC transmission as  $K$  becomes large. On the one hand, for the expected throughput, transmission delay is not an issue and hence the optimal strategy is to select only a subset of users with the best joint channel realizations for transmission at one time. The expected throughput thus scales linearly with  $M$  and in double-logarithm with  $K$ , for which the former is due to spatial multiplexing gain and the latter arises from the multiuser-diversity gain. On the other hand, in the delay-limited case, a constant-rate transmission needs to be ensured for all users. Consequently, as the number of users increases, though more degrees of freedom are available for optimizing transmit parameters such as precoding vectors and encoding order of users, the delay-limited throughput is eventually saturated. The above comparison demonstrates that for a SDMA-based network with a large user population, transmission delay can be a critical factor that prevents from achieving the maximum asymptotic throughput. However, notice that this may not be the case for the network having similar  $M$  and  $K$ , as will be verified later by the simulation results.

## V. SIMULATION RESULTS

In this section, simulation results are presented for evaluating the performances of dynamic resource allocation for the fading MISO-BC under various transmission delay considerations. Since the network throughput is contingent on transmit delay requirements as well as many other factors such as number of transmit antennas at the BS, number of mobile users, user channel conditions and rate requirements, and the transmit power constraint at the BS, various combinations of these factors are considered in the following simulations with an aim to demonstrate how transmission delay interplays with other factors in determining the achievable network throughput. The simulation results are presented in the following subsections. Note that in the following simulation, user channel vectors  $\{\mathbf{h}_k(n)\}$  are independently generated from the population of CSCG vectors, and if not stated otherwise, it is assumed that  $\mathbf{h}_k(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \forall k$ .

### A. Transmit Optimization for Mixed NDC and DC Traffic

First, consider a MISO-BC with  $M = K = 4$  with two users having NDC transmission and the other two having DC transmission. For convenience, it is assumed that the target average transmit rates for the two users with NDC transmission are both equal to  $R_{\text{NDC}}^*$ , and the target constant rates for the two users with DC transmission both equal to  $R_{\text{DC}}^*$ . Let  $\gamma$  denote the *loading factor* representing the ratio of the total amount of NDC traffic to that of the sum of NDC and DC traffic, i.e.,  $\gamma \triangleq \frac{R_{\text{NDC}}^*}{R_{\text{NDC}}^* + R_{\text{DC}}^*}$ . If the proposed algorithm in Table I that achieves optimal dynamic resource allocation is used, the required average transmit power at the BS is expected to be the minimum for satisfying both NDC and DC user rate demands for any given  $\gamma$ . For purpose of comparison, two suboptimal transmission schemes are also considered:

- *Time-Division-Multiple-Access (TDMA)*: A simple transmission scheme is to divide each transmission period into  $K$  consecutive equal-duration time-blocks, each dedicated for transmission of one user's data traffic. If coherent precoding is applied at each fading state  $n$ , i.e., the precoder for user  $k$ , defined as  $\hat{\mathbf{b}}_k(n) \triangleq \frac{\mathbf{b}_k(n)}{\|\mathbf{b}_k(n)\|}$ , is equal to  $\frac{\mathbf{h}_k^\dagger(n)}{\|\mathbf{h}_k(n)\|}, \forall k$ , it can be shown that the MISO-BC is decomposable into  $K$  single-user SISO channels. Depending on each user's delay requirement, conventional water-filling power control [24] and channel-inversion power control [27] can be applied over different fading states for users with NDC and DC transmission, respectively, to achieve the minimum average transmit power under the given user rate demand.

- *Zero-Forcing (ZF) -Based SDMA*: Another possible transmission scheme is also based on the SDMA principle, i.e., supporting all user's transmission simultaneously by multi-antenna spatial multiplexing at the BS. However, instead of using the dirty-paper-coding (DPC)-based non-linear precoding assumed in this paper, a simple ZF-based linear precoding is employed at the BS. The ZF-based precoder  $\hat{\mathbf{b}}_k(n)$  for user  $k$  at each fading state  $n$  maximizes the user's own equivalent channel gain  $\|\mathbf{h}_k(n)\hat{\mathbf{b}}_k(n)\|^2$  subject to the constraint that its associated co-channel interference must be completely removed, i.e.,  $\mathbf{h}_{k'}(n)\hat{\mathbf{b}}_k(n) = 0, \forall k' \neq k$ . Like TDMA, ZF-based precoding also decomposes the MISO-BC into  $K$  (assuming  $K \leq M$ ) single-user SISO channels, and thereby optimal single-user power control schemes can be applied.

In Fig. 5 and Fig. 6, these three schemes are compared for two cases of network throughput, one corresponding to the sum-rate of users  $2R_{\text{NDC}}^* + 2R_{\text{NDC}}^* = 6$  bits/complex dimension, and one corresponding to 2 bits/complex dimension. The required average transmit power over 500 randomly generated channel realizations is plotted versus the loading factor  $\gamma$ . First, in Fig. 5 it is observed that in the case of high throughput (bandwidth-limited), ZF-based SDMA outperforms TDMA because of its larger spectral efficiency by spatial multiplexing. However, in the case of low throughput (power-limited), in Fig. 6 it is observed that TDMA achieves better power efficiency than ZF-based SDMA. This is because coherent precoding in TDMA provides more diversity and array gains, which become more dominant over spatial multiplexing gains at the power-limited regime. Secondly, it is observed that in both cases of high and low throughput, the proposed scheme always outperforms both TDMA and ZF-based SDMA given any loading factor  $\gamma$ . This is because the proposed scheme optimally balances the achievable spatial multiplexing, array, and diversity gains for the fading MISO-BC. Thirdly, it is observed that for all schemes, the required transmit power associated with some  $\gamma$ ,  $\gamma < 0.5$ , is always larger than that with  $1 - \gamma$  (e.g., comparing  $\gamma = 0.1$  and  $\gamma = 0.9$ ), i.e., given the same portion of data traffic in the total traffic, NDC traffic has a better power efficiency than DC traffic. This is because NDC traffic allows for more flexible dynamic resource allocation than DC traffic and thus leads to a better power efficiency.

### B. Convergence of Online Algorithm

The convergence of the online algorithm in Table II is validated by simulations. For simplicity, it is assumed that only NDC users are present in the network. A MISO-BC with  $K = 2$  and  $M = 4$  is considered. The target average-rates for user-1 and user-2 are 3 and 1 bits/complex dimension, respectively. The online algorithm is implemented for 3000 consecutive transmissions with randomly generated channel realizations. Initially, the dual

variables are set as  $\mu_1[0] = \mu_2[0] = 1$ , and the estimated average transmit rates as  $\bar{R}_1[0] = \bar{R}_2[0] = 0$ . The updates for  $\{\bar{R}_k[t]\}$  and  $\{\mu_k[t]\}$  as  $t$  proceeds are shown in Fig. 7 and Fig. 8, respectively, for  $\Delta = \epsilon = 0.01$ . It is observed that the online algorithm converges to the optimal dual variables and target average-rates for both users after a couple of hundreds of iterations. Simulations results (not shown in this paper) for other values of  $\Delta$  and  $\epsilon$  indicate that in general a larger step size leads to a faster algorithm convergence but also results in more frequent oscillations.

### C. Throughput-Delay Tradeoff

Fig. 9 compares the network throughput under two extreme cases of delay constraint considered in this paper, namely, the expected throughput and the delay-limited throughput. It is assumed that  $M = 2$ , and two types of networks are considered: One is a 2-user network with user rate profile  $\alpha = [\frac{2}{3} \ \frac{1}{3}]$ ; and the other is a 4-user network with  $\alpha = [\frac{2}{6} \ \frac{2}{6} \ \frac{1}{6} \ \frac{1}{6}]$ . Notice that the ratio of rate demand between any of the first two users and any of the last two users in the second case is the same as that between the two users in the first case, which is equal to 2. First, it is observed that for both networks, the delay penalties are only moderate for all considered transmit power values. The delay penalty increases as  $K$  becomes larger than  $M$ , but only slightly. Small delay penalties in both cases can be explained by extending the multi-antenna channel hardening effect [28] in the single-user case to the fading MISO-BC, i.e., as the number of degrees of fading increases with  $M$  for some constant  $K$ , not only the mutual information associated with each user's channel, but also the whole capacity region of the BC becomes less variant over different fading states. Therefore, the sum-rate of users for any given rate profile also changes less dramatically, and as a consequence, imposing a set of strict constant-rate constraints at each fading state (equivalent to a fixed rate-profile) does not incur a large throughput loss for the fading MISO-BC.

Secondly, it is observed that the expected throughput for a 4-user network outperforms that for a 2-user network given that both networks have the similar allocated rate portion among users. This throughput gain can be explained by the well-known multiuser diversity effect [13] for NDC transmission. As the number of degrees of fading increases with  $K$  for some constant  $M$ , the BS can select relatively fixed number of users (around  $M$ ) with the best joint channel realizations from a larger number of total users for transmission at each time. Thereby, the network throughput is boosted provided that each user has sufficient delay tolerance. On the other hand, it is observed that, maybe more surprisingly, the delay-limited throughput for the 4-user network also outperforms that for the 2-user network under the similar allocated rate portion among users. Notice that in this case, all

user's rates need to be constant at each fading state, and consequently, adaptive rate allocation for achieving the expected throughput does not apply here. This throughput gain in the DC case can also be explained by a more generally applied multiuser diversity effect. Even though a set of constant-rates of users need to be satisfied at each fading state in the DC case, a larger number of users provides the BS more flexibility in jointly optimizing allocation of transmit resources among users based on their channel realizations. This multiuser diversity gain becomes more substantial as  $K$  increases because a new user can bring along additional  $M$  degrees of fading. However, it is also important to take note that the multiuser diversity gain in the delay-limited case does have certain limitation, e.g., as  $K$  becomes overwhelmingly larger than  $M$ , the delay-limited throughput eventually gets saturated (see Theorem 4.1).

#### D. Throughput-Fairness Tradeoff

At last, the tradeoff between the achievable network throughput and the fairness for user rate allocation is demonstrated. The NDC transmission is considered and hence the expected throughput is of interest. A network with 2 users is considered and it is assumed that  $M = 2$ , and the average LTPC at the BS is fixed as 10. Two fading channel models are considered: One has symmetric fading where both user channel vectors  $\mathbf{h}_k(n)$ ,  $k = 1, 2$ , are assumed to be distributed as  $\mathcal{CN}(\mathbf{0}, \mathbf{I})$ ; and the other has asymmetric fading where  $\mathbf{h}_1(n) \sim \mathcal{CN}(\mathbf{0}, 2\mathbf{I})$ , and  $\mathbf{h}_2(n) \sim \mathcal{CN}(\mathbf{0}, \frac{1}{2}\mathbf{I})$ . Notice that the asymmetric-fading case may correspond to a near-far situation in the cellular network where user-1 is closer to the BS and hence has an average channel gain of approximately  $20 \times \log_{10} 4 = 12$  dB compared to user-2. Let  $\phi$  denote the ratio of average-rate demand between user-1 and user-2, i.e., for the corresponding rate profile  $\alpha$ ,  $\frac{\alpha_1}{\alpha_2} = \phi$ . In Fig. 10, the expected throughput is shown as a function of  $\phi$  for both symmetric- and asymmetric-fading cases. It is observed that in the symmetric-fading case, a strict fairness constraint for equal rate allocation among users, i.e.,  $\phi = 0.5$ , also corresponds to the maximum expected throughput or the sum capacity. In contrast, for the case of asymmetric fading, the maximum expected throughput is achieved when  $\phi = 0.7$ , i.e., user-1 is allocated 70% of the expected throughput because of its superior channel condition. However, in the latter case, a strict fairness constraint with  $\phi = 0.5$  yields a throughput loss of only 0.3 bits/complex dimension. This small fairness penalty can be explained by observing that the expected throughput for the fading MISO-BC under both symmetric and asymmetric fading is quite insensitive to  $\phi$  at a very large range of its values, indicating that by optimizing resource allocation, transmission with very heterogeneous rate requirements may incur only a negligible network throughput loss.

## VI. CONCLUDING REMARKS

This paper investigates the capacity limits and the associated optimal dynamic resource allocation schemes for the fading MISO-BC under various transmission delay and fairness considerations. First, this paper studies transmit optimization with mixed delay-constrained and delay-tolerant data traffic. The proposed online resource allocation algorithm is based on a two-layer Lagrange-duality method, and is amenable to real-time cross-layer implementation. Secondly, this paper characterizes some key fundamental tradeoffs between network throughput, transmission delay and user fairness in rate allocation, and draws some novel insights pertinent to multiuser diversity and channel hardening effects for the fading MISO-BC. This paper shows that when there are similar numbers of users and transmit antennas at the BS, the delay penalty and the fairness penalty in the achievable network throughput may be only moderate, suggesting that employing multi-antennas at the BS is an effective means for delivering data traffic with heterogeneous delay and rate requirements.

The results obtained in this paper can also be extended to the uplink transmission in a cellular network by considering the fading MAC under individual user power constraint instead of the sum-power constraint in this paper as a consequence of the BC-MAC duality. Hence, another important factor needs to be taken into account by dynamic resource allocation for the fading MAC is the fairness in user transmit power consumption. The concept of rate profile in this paper can also be applied to define a similar power profile for regulating the power consumption between users for the MAC [11]. Furthermore, although the developed results in this paper are under the assumption of capacity-achieving transmission using DPC-based non-linear precoding at the BS, they are readily extendible to other suboptimal transmission methods such as linear precoding provided that the achievable rate region by these methods is still a convex set and, hence, like in this paper similar convex optimization techniques can be applied.

## APPENDIX I

### ALTERNATIVE ALGORITHM FOR PROBLEM P3

In Problem P3, the constraints are separable and the objective function is convex. Hence, this problem can be solved iteratively by the block-coordinate decent method [21]. At each iteration, this method minimizes the objective function w.r.t. one  $q_k$  while holding all the other  $q_k$ 's constant. More specifically, the method minimizes (23) w.r.t.  $q_1$  with constant  $\{q_2, \dots, q_K\}$ , and then  $q_2$  with constant  $\{q_1, q_3, \dots, q_K\}$ , ..., to  $q_K$  with constant  $\{q_1, \dots, q_{K-1}\}$ , and after that the above routine is repeated. Because after each iteration the objective function

only decreases, the convergence to the global minimum of the objective function is ensured. Each iteration of the above algorithm is described as follows. Without loss of generality, assuming that in (23),  $\pi(k) = k, k = 1, \dots, K$ . Considering any arbitrary iteration for minimizing (23) w.r.t.  $q_m, m \in \{1, \dots, K\}$ , with all the other  $q_k$ 's constant, Problem P3 can be rewritten as

$$\text{Minimize} \quad q_m - \sum_{k=m}^K (\beta_k - \beta_{k+1}) \log_2 \left| \mathbf{h}_m^\dagger \mathbf{h}_m q_m + \sum_{i=1, i \neq m}^k \mathbf{h}_i^\dagger \mathbf{h}_i q_i + \mathbf{I} \right| \quad (41)$$

$$\text{Subject to} \quad q_m \geq 0. \quad (42)$$

By introducing the dual variable  $\lambda_m, \lambda_m \geq 0$ , associated with the constraint  $q_m \geq 0$ , the Karush-Kuhn-Tacker (KKT) optimality conditions [20] state that the optimal  $p_m^*$  and dual variable  $\lambda_m^*$  for this problem must satisfy

$$\sum_{k=m}^K (\beta_k - \beta_{k+1}) \mathbf{h}_m \left( \mathbf{h}_m^\dagger \mathbf{h}_m q_m^* + \sum_{i=1, i \neq m}^k \mathbf{h}_i^\dagger \mathbf{h}_i q_i + \mathbf{I} \right)^{-1} \mathbf{h}_m^\dagger = (1 - \lambda_m^*) \log 2, \quad (43)$$

$$\lambda_m^* q_m^* = 0. \quad (44)$$

Let  $d(q_m^*)$  denote the function on the left-hand-side of (43). From (44), it is inferred that  $q_m^* > 0$  only if  $\lambda_m^* = 0$ . From (43) and by taking note that  $d(q_m^*)$  is a non-increasing function of  $q_m^*$  for  $q_m^* \geq 0$ , it follows that  $q_m^* > 0$  occurs only if  $d(0) > \log 2$ . Thus, it follows that

$$q_m^* = \begin{cases} 0 & \text{if } d(0) \leq \log 2 \\ q_0 & \text{otherwise,} \end{cases} \quad (45)$$

where  $q_0$  is the unique root for  $d(q_m^*) = \log 2$ .  $q_0$  can be easily found by a bisection search [20] over  $[0, \frac{\beta_m}{\log 2}]$ , where the above upper-bound for  $q_0$  is obtained by the following inequalities and equality:

$$\log 2 \leq \sum_{k=m}^K (\beta_k - \beta_{k+1}) \mathbf{h}_m \left( \mathbf{h}_m^\dagger \mathbf{h}_m q_m^* + \mathbf{I} \right)^{-1} \mathbf{h}_m^\dagger \quad (46)$$

$$\leq \frac{1}{q_m^*} \sum_{k=m}^K (\beta_k - \beta_{k+1}) \quad (47)$$

$$= \frac{\beta_m}{q_m^*}. \quad (48)$$

## APPENDIX II

### PROOF OF THEOREM 4.1

First, we obtain an upper-bound for the delay-limited throughput  $C_d$  by assuming that there is no co-channel interference between users, as opposed to the successive interference pre-subtraction by DPC. Under this assumption, the fading MISO-BC is decomposed into  $K$  parallel single-user fading channels. Let  $\hat{p}_k$  denote the average

transmit power assigned to user  $k$ ,  $k = 1, \dots, K$ . The maximum constant-rate achievable over all fading states (or the so-called delay-limited capacity [4]) for user  $k$  can be expressed as [27]

$$C_d(k) = \log_2 \left( 1 + \frac{\hat{p}_k}{\rho_k} \right), \quad (49)$$

where  $\rho_k = \mathbb{E}_n \left[ \frac{1}{\|\mathbf{h}_{k(n)}\|^2} \right]$ . Because of the assumed symmetric fading (hence,  $\rho_k = \rho, \forall k$ ) and symmetric rate demand (hence,  $C_d(k) = \frac{C_d}{K}, \forall k$ ), it follows that  $\hat{p}_k = \frac{p^*}{K}, \forall k$ , achieves the maximum average sum-rate of users. Hence, the delay-limited throughput is upper-bounded by

$$C_d \leq K \log_2 \left( 1 + \frac{p^*}{K\rho} \right), \quad (50)$$

which holds for any  $K \geq 1$ . By taking the limit of the right-hand-side of (50) as  $K \rightarrow \infty$ , the proof of Theorem 4.1 is completed.

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```

Initialize  $\{\mu_k\}, k \in \mathcal{U}_{\text{NDC}}$  and  $\{\delta_k(n)\}, k \in \mathcal{U}_{\text{DC}}$ 
While not all  $\mathbb{E}_n[R'_k(n)]$  converges to  $R_k^*, k \in \mathcal{U}_{\text{NDC}}$ , do
  For  $n = 1, 2, \dots$  do
    While not all  $R'_k(n)$  converges to  $R_k^*, k \in \mathcal{U}_{\text{DC}}$ , do
      Solve Problem P3 for fading state  $n$  to obtain  $\{R'_k(n)\}$ 
      Update  $\{\delta_k(n)\}$  according to (28).
    End While
  End For
  Update  $\{\mu_k\}$  according to (27).
End While

```

TABLE I

PROPOSED ALGORITHM FOR PROBLEM P1.

```

Initialize  $\{\mu_k[0]\}, k \in \mathcal{U}_{\text{NDC}}$ .
Set  $\bar{R}_k[0] = 0, \forall k \in \mathcal{U}_{\text{NDC}}; t = 1$ .
Repeat
   $\mu_k[t] \leftarrow \{\mu_k[t-1] + \Delta(R_k^* - \bar{R}_k[t-1])\}^+, k \in \mathcal{U}_{\text{NDC}}$ .
  Initialize  $\{\delta_k[t]\}, k \in \mathcal{U}_{\text{DC}}$ .
  While not all  $R_k[t]$  converges to  $R_k^*, k \in \mathcal{U}_{\text{DC}}$ , do
    Solve Problem P3 for given  $\{\mu_k[t]\}, \{\delta_k[t]\}$  and  $\{\mathbf{h}_k^\dagger[t]\}$  to obtain  $\{R_k[t]\}$ .
     $\delta_k[t] \leftarrow \{\delta_k[t] + \Delta(R_k^* - R_k[t])\}^+, k \in \mathcal{U}_{\text{DC}}$ .
  End While
   $\bar{R}_k[t] = (1 - \epsilon)\bar{R}_k[t-1] + \epsilon R_k[t]$ .
   $t \leftarrow t + 1$ .

```

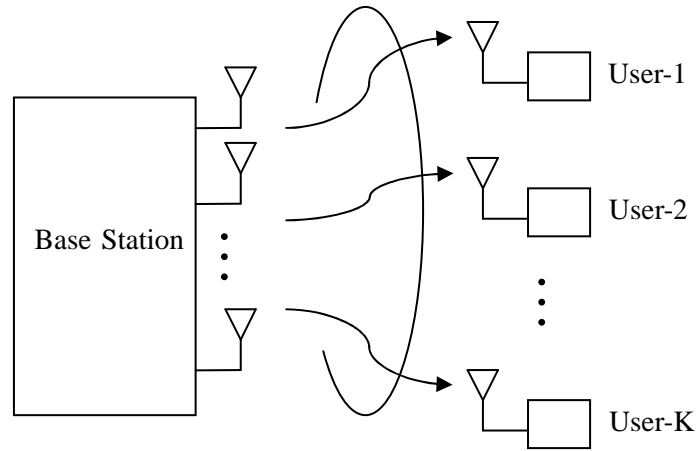


Fig. 1. MISO-BC for SDMA-based downlink transmission in a single-cell of wireless cellular network.

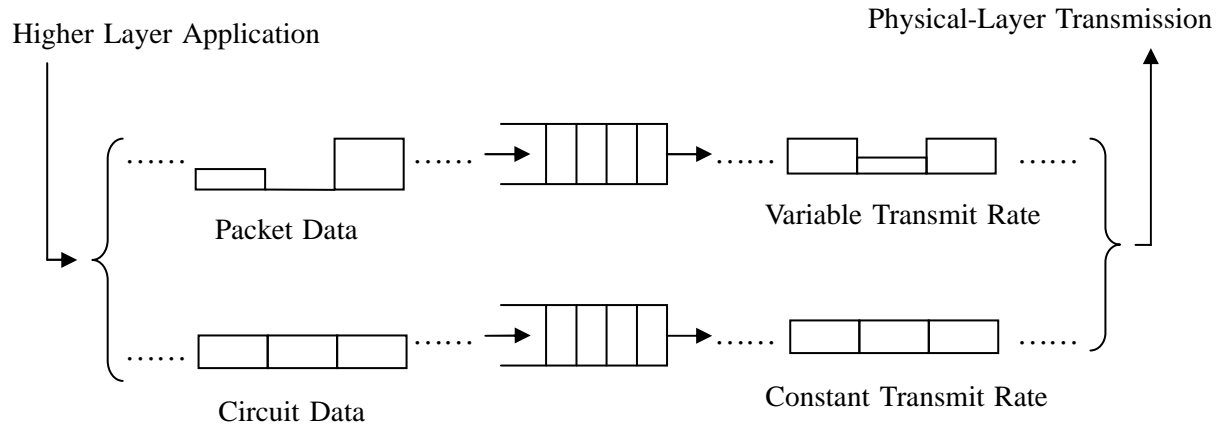


Fig. 2. Transmission scheduling at the MAC-layer for packet data and circuit data.

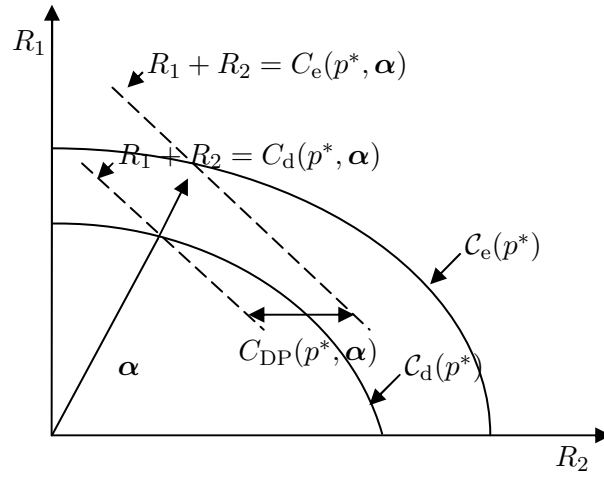


Fig. 3. Illustration of the delay penalty  $C_{DP}(p^*, \alpha)$ .

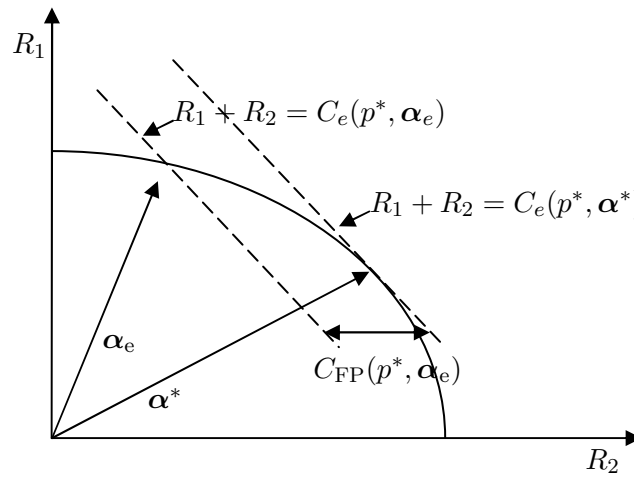


Fig. 4. Illustration of the fairness penalty  $C_{FP}(p^*, \alpha_e)$  in the expected capacity region  $\mathcal{C}_e(p^*)$ .

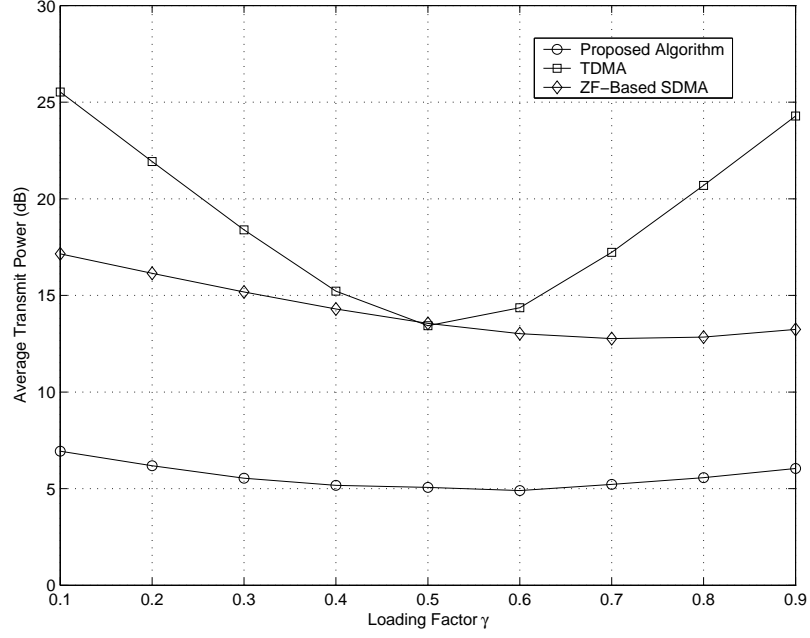


Fig. 5. Comparison of the average transmit powers for different schemes under mixed NDC and DC transmission. The total amount of NDC and DC traffic is 6 bits/complex dimension.

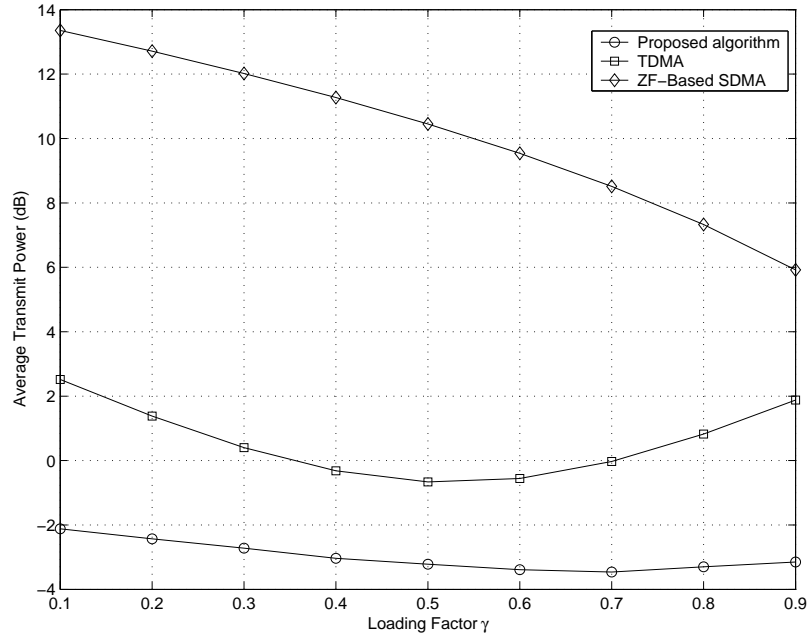


Fig. 6. Comparison of the average transmit powers for different schemes under mixed NDC and DC transmission. The total amount of NDC and DC traffic is 2 bits/complex dimension.

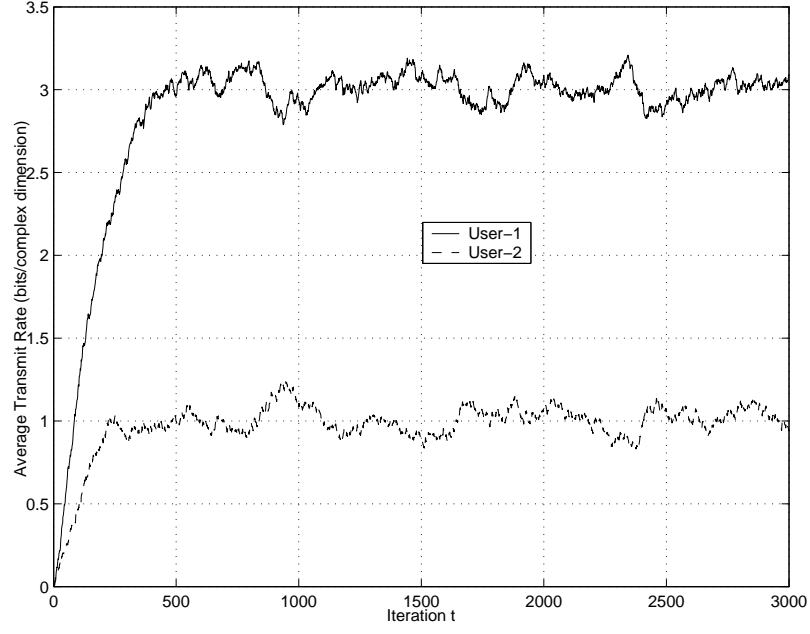


Fig. 7. The estimated average transmit rate  $\bar{R}_k[t]$  at different time  $t$  obtained by the online algorithm. The target average-rate for user-1 and user-2 are 3 and 1 bits/complex dimension, respectively.

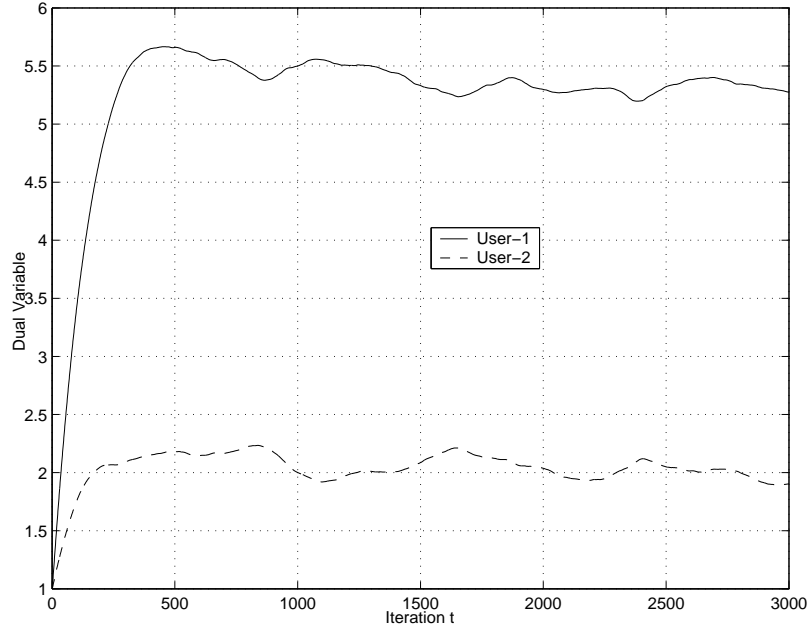


Fig. 8. The dual variable  $\mu_k[t]$  at different time  $t$  obtained by the online algorithm.

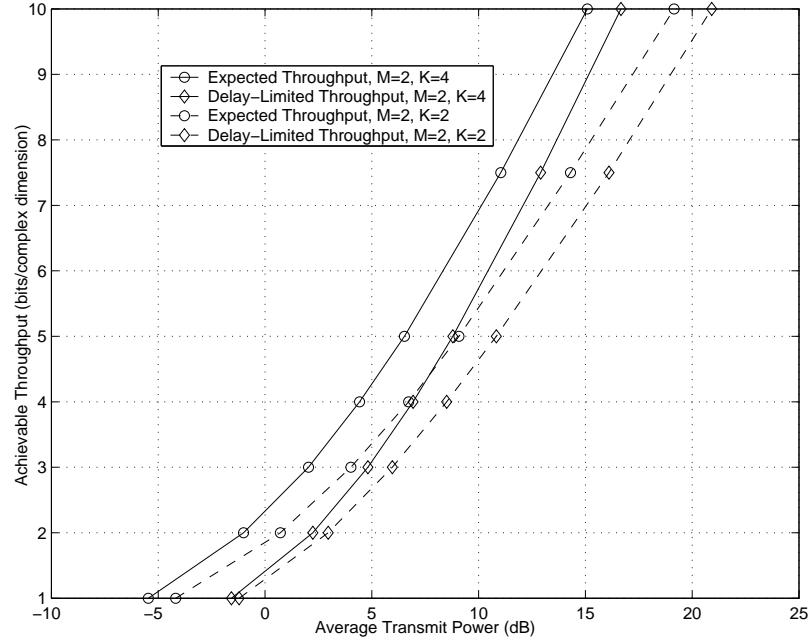


Fig. 9. Comparison of the expected throughput and the delay-limited throughput.

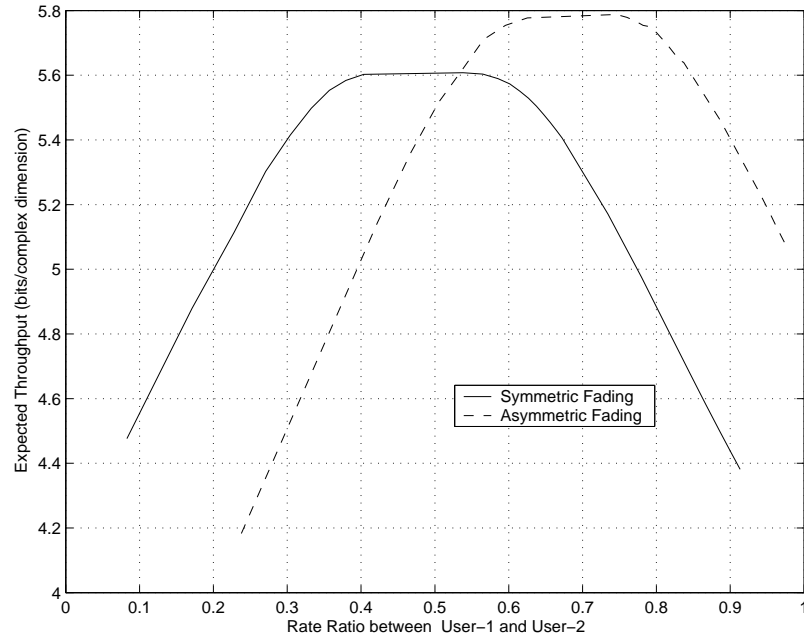


Fig. 10. Comparison of the expected throughput under different fairness constraints.